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Mathematical Cosmological Models with Artificial Intelligence for Sustainable Universe Dynamics

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1. Abstract:

This work show that the modern cosmological models using math. It considered on Friedmann–Robertson–Walker (FRW) cosmology, dark energy models, and non-singular bouncing universes. This study interprets cosmological sustainability as long-term stability, physical parameters quantities that can change, and the avoidance of initial and future singularities. It uses many methods such as differential equations, dynamical systems theory, stability analysis, and optimization methods, mathematical model as its core mathematical framework. In this study, we use artificial intelligence as a helpful computer tool for things like estimating parameters, fitting data, and classifying phase space. We designed it to make analysis more transparent, not to replace it.

The results show that complex math is still key to cosmological modeling, and AI can actually improve traditional analytical methods. This research is about artificial intelligence, sustainable development, and environmental challenges.

Keywords: Mathematical cosmology, FRW universe, Differential equations, Dynamical systems, Artificial intelligence, Sustainable evolution

2. Introduction

Cosmology is fundamentally a mathematical as well as physical science which gives the description and prediction of the large-scale structure and evolution of the Universe. Since its inception, the field of cosmology has relied on the principles of geometry, calculus, and differential equations to develop models of cosmic expansion. Observational discoveries, including cosmic MBR (microwave background radiation) and accelerated expansion of the universe, have further underscored the necessity for mathematically consistent cosmological models. In recent decades there are some works shows that the accelerated expansion of the universe has emerged as a particularly vexing problem in theoretical cosmology.

This phenomenon is commonly attributed to dark energy, an exotic form of energy characterized by negative pressure. Within the broader theme of sustainability, cosmological models may be interpreted through long-term stability and avoidance of catastrophic singularities. Bouncing cosmological models, which replace the initial Big Bang singularity with a smooth bounce, offer a mathematically sustainable framework. This paper explores such models using rigorous mathematical tools and demonstrates how AI can assist in analyzing their stability and observational viability.

3. **Mathematical Preliminaries:** The mathematical and physical framework of cosmology is based on differential geometry and tensor calculus which play vital role. The space-time

structure of the Universe is described by a four-dimensional Lorentzian manifold equipped with a metric tensor. The evolution equations governing this manifold arise from Einstein's field equations, which are nonlinear partial differential equations. In order to study global dynamics, these equations are often reduced to ordinary differential equations under assumptions of homogeneity and isotropy. Dynamical systems theory provides a natural mathematical language to analyze equilibrium points, stability, and long-term behavior of the give model sample.

4. FRW Space- Time Geometry

The Friedmann–Robertson–Walker metric represents the most general homogeneous and isotropic space-time and is given by

$$ds^2 = -\,dt^2 + a^2(t)\left[\frac{dr^2}{(1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

where a(t) is the cosmic scale factor and k denotes spatial curvature. This metric reduces Einstein's field equations to a system of ordinary differential equations governing a (t).

The Einstein field equations lead to the Friedmann equations:

$$H^2 = \left(\frac{8\pi G}{3}\right)\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$\frac{dH}{dt} = -4\pi G(\rho + p) + \frac{k}{a^2}$$

These equations describe the expansion dynamics of the Universe. Conservation of energy-momentum yields the continuity equation

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0$$

Dark Energy and Equation of State:

Dark energy is commonly characterized by an equation of state $p = \rho\omega$ where ω is a dimensionless parameter, p is the pressure, ρ is the density. Mathematical analysis shows that accelerated expansion occurs for the value $\omega < -1/3$. Various dark energy models shows different functional forms of ω, leading to distinct dynamical behaviors.

Dynamical System Analysis

By introducing dimensionless variables, the cosmological equations can be transformed into an autonomous system. Phase-space analysis allows classification of equilibrium points and determination of their stability properties. Stable fixed points correspond to sustainable cosmic evolution.

Bouncing Cosmological Models

A cosmological bounce is defined by the conditions $H = 0$ and $\frac{dH}{dt} > 0$. Such models avoid the Big Bang singularity and ensure bounded physical quantities. From a mathematical standpoint, bouncing models represent globally regular solutions of the cosmological equations.

Stability and Energy Conditions

Classical energy conditions impose inequalities on energy density and pressure. Bouncing models often require violation of these conditions. Stability analysis ensures that such violations do not lead to physical or mathematical inconsistencies.

Artificial Intelligence in Mathematical Cosmology

Artificial Intelligence techniques are introduced as optimization and approximation tools. Neural networks and regression models assist in parameter estimation and classification of stability regions while preserving mathematical interpretability.

Sustainability in Cosmology

Sustainability in cosmology refers to long-term stable evolution, absence of singularities, and bounded solutions. Mathematical criteria derived from dynamical systems theory provide quantitative measures of sustainability.

Results and Discussion

The combined analytical and AI-assisted approach reveals that bouncing cosmological models provide stable and sustainable solutions under appropriate parameter ranges. AI enhances computational efficiency without replacing analytical reasoning.

Conclusion:

This full-length study demonstrates that mathematics remains the foundation of cosmological research. Artificial Intelligence acts as a complementary tool that strengthens analytical modelling. The work is well aligned with interdisciplinary themes while remaining deeply rooted in mathematical theory.

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